Unit 3

Matrices

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. If matrix A is of order $m \times n$, and for matrix B, AB and BA both are defined, then order of matrix B is

(a) $m \times n$ (b) $n \times n$ (c) $m \times m$ (d) $n \times m$

Ans: (d) $n \times m$

2. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, then the value of k if, $A^2 = kA - 2I$ is

(a) 0 (b) 8 (c) – 7 (d) 1

Ans: (d) 1

3. If $\begin{bmatrix} 3 & 4 \\ 2 & x \end{bmatrix}\begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 19 \\ 15 \end{bmatrix}$ then the value of x is

(a) 3 (b) 2 (c) 5 (d) 1

Ans: (c) 5

4. For what value of x, is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew symmetric matrix?

(a) 0 (b) 2 (c) –2 (d) –3

Ans: (b) 2

5. For the matrix $X = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $(X^2 - X)$ is

(a) 2I (b) 3I (c) I (d) 5I

Ans: (a) 2I

6. If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A = A^T$, where A^T is the transpose of the matrix A, then

(a) x = 0, y = 5 (b) x = y (c) x + y = 5 (d) x = 5, y = 0

Ans: (b) x = y

7. If $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $A = B^2$, then x equals:
   (a) ± 1 (b) – 1 (c) 1 (d) 2
   Ans: (c) 1

8. Number of symmetric matrices of order 3 × 3 with each entry 1 or – 1 is
   (a) 512 (b) 64 (c) 8 (d) 4
   Ans. (b) 64
   Number of Symmetric matrices of order 3 × 3 = 2^6 = 64

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** If $\begin{bmatrix} x & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -4 & 0 \end{bmatrix} = 0$, then x = 2.

   Reason (R): If $\begin{bmatrix} x & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -4 & 0 \end{bmatrix} = 0$, then x = 4.

   Ans: (d) A is false but R is true.

   $$\begin{bmatrix} x & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x-8 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

   ⇒ 2x – 8 = 0 (By definition of equality)
   ⇒ x = 4

10. **Assertion (A):** If the order of A is 3 × 4, the order of B is 3 × 4 and the order of C is 5 × 4, then the order of $(A^TB)C^T$ is 4 × 5.
    Reason (R): To multiply an m × n matrix by n × p matrix the n must be the same and result is an m × p matrix. Also, A be a matrix of order m × n then the order of transpose matrix is n × m.
    Ans. (a) Both A and R are true and R is the correct explanation of A.
    Order of A = 3 × 4
    Order of B = 3 × 4
    and Order of C = 5 × 4
    The, Order of A^T = 4 × 3
    and Order of C^T = 4 × 5
    Now, Order of (A^TB) = 4 × 4
    Thus, Order of $(A^TB)C^T$ = 4 × 5

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that F(x) F(y) = F(x + y).

    Ans:

    $$LHS = F(x)F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x \cos y - \sin x \sin y & -\sin y \cos x - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(x+y) = RHS$$

12. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find $(x - y)$.

Ans:

Given that $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

Equating we get $8 + y = 0$ and $2x + 1 = 5$

$\Rightarrow y = -8$ and $x = 2$

$\Rightarrow x - y = 2 + 8 = 10$

13. Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$?

Ans:

Given that $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$

Since Matrix multiplication is associative, therefore $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} x+0+2 \\ 0+8+1 \\ 2x+0+3 \end{bmatrix} = O$

$\Rightarrow \begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} x+2 \\ 9 \\ 2x+3 \end{bmatrix} = O$

$\Rightarrow \begin{bmatrix} x(x+2) + (-5).9 + (-1)(2x+3) \end{bmatrix} = O$

$\Rightarrow \begin{bmatrix} x^2 - 48 \end{bmatrix} = O \Rightarrow x^2 - 48 = 0 \Rightarrow x^2 = 48$

$\Rightarrow x = \pm\sqrt{48} = \pm 4\sqrt{3}$

14. If A and B are symmetric matrices, show that $AB + BA$ is symmetric and $AB - BA$ is skew symmetric.

Ans: $A' = A$, $B' = B$;

Consider $(AB + BA)' = (AB)' + (BA)' = B'A' + A'B' = BA + AB = AB + BA$.

Hence, symmetric.

Consider $(AB - BA)' = (AB)' - (BA)' = B'A' - A'B'$

$BA - AB = -(AB - BA)$
Hence, skew symmetric.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find $A^2 - 5A + 6I$, if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Ans:

$$A^2 = A.A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$\therefore A^2 - 5A + 6I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10+6 & -1-0+0 & 2-5+0 \\ 9-10+0 & -2-5+6 & 5-15+0 \\ 0-5+0 & -1+5+0 & -2+0+6 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

16. Find the matrix X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & 3 \\ 2 & 4 & 6 \end{bmatrix}$

Ans:

Given that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & 3 \\ 2 & 4 & 6 \end{bmatrix}$

The matrix given on the RHS of the equation is a 2×3matrix and the one given on the LHS of the equation is as a 2×3 matrix. Therefore, X has to be a 2×2 matrix.

Now, let $X = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

$$\begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & 3 \\ 2 & 4 & 6 \end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have
$a + 4c = -7$, $\quad 2a + 5c = -8$, $3a + 6c = -9$
$b + 4d = 2$, $\quad 2b + 5d = 4$, $\quad 3b + 6d = 6$
Now, $a + 4c = -7 \Rightarrow a = -7 - 4c$
$2a + 5c = -8 \Rightarrow -14 - 8c + 5c = -8$
$\Rightarrow -3c = 6 \Rightarrow c = -2$
$\therefore a = -7 - 4(-2) = -7 + 8 = 1$
Now, $b + 4d = 2 \Rightarrow b = 2 - 4d$ and $2b + 5d = 4 \Rightarrow 4 - 8d + 5d = 4$
$\therefore -3d = 0 \Rightarrow d = 0$
$\therefore b = 2 - 4(0) = 2$
Thus, $a = 1, b = 2, c = -2, d = 0$

Hence, the required matrix X is $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

17. Express the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

Ans: Let $A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}, then \;\; A=P+Q$

$where, \;\; P=\frac{1}{2}(A+A') \;\; and \;\; Q=\frac{1}{2}(A-A')$

$$Now, P=\frac{1}{2}(A+A')=\frac{1}{2}\left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}+\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\therefore P'=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}=P$$

$Thus \;\; P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$Now, Q=\frac{1}{2}(A-A')=\frac{1}{2}\left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}-\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore Q'=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}=-Q$$

$Thus \;\; Q=\frac{1}{2}(A-A')$ is a skew symmetric matrix.

Representing A as the sum of P and Q,

$$P+Q=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}+\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}=A$$

SECTION – D

Questions 18 carry 5 marks.

18. If A = $\begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of the equations :

$x + 2y – 3z = 6, 3x + 2y – 2z = 3, 2x – y + z = 2$

Ans:

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{vmatrix} = 1(2-2) - 2(3+4) - 3(-3-4) = -14 + 21 = 7 \neq 0$$

$\therefore$ A^{-1} exists

Now, $A_{11} = 0, A_{12} = -7, A_{13} = -7, A_{21} = 1, A_{22} = 7,$
$A_{23} = 5, A_{31} = 2, A_{32} = -7, A_{33} = -4$

$$\therefore \text{ adj } A = \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

The given system of equations is
$$x + 2y - 3z = 6$$
$$3x + 2y - 2z = 3$$
$$2x - y + z = 2$$

The system of equations can be written as $AX = B$

$$\text{where } A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$\because$ A^{-1} exists, so system of equations has a unique solution given by $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$$

$\Rightarrow x = 1, y = -5, z = -5$

OR

The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

Ans: Let the first , second and third number be x, y, z respectively.

Then, according to the given condition, we have

$x + y + z = 6$

$y + 3z = 11$

$x + z = 2y$ or $x - 2y + z = 0$

This system of equations can be written as AX = B, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \ \& \ B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$A = 1(1 + 6) - 0 + 1(3 - 1) = 9$

$\Rightarrow |A| \neq 0$

$\therefore$ The system of equation is consistent and has a unique solution.

Now, we find adj(A)

$A_{11} = 7, A_{12} = 3, A_{13} = -1,$

$A_{21} = -3, A_{22} = 0, A_{23} = 3,$

$A_{31} = 2, A_{32} = -3, A_{33} = 1$

Hence, $adj(A)=\begin{bmatrix}7 & -3 & 2\\ 3 & 0 & -3\\ -1 & 3 & 1\end{bmatrix}$

Thus, $A^{-1}=\frac{1}{|A|}adj(A)\ =\frac{1}{9}\begin{bmatrix}7 & -3 & 2\\ 3 & 0 & -3\\ -1 & 3 & 1\end{bmatrix}$

Since, AX=B

$\therefore X = A^{-1}B$

$\Rightarrow X=\frac{1}{9}\begin{bmatrix}7 & -3 & 2\\ 3 & 0 & -3\\ -1 & 3 & 1\end{bmatrix}\begin{bmatrix}6\\ 11\\ 0\end{bmatrix} \Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{9}\begin{bmatrix}9\\ 18\\ 27\end{bmatrix}=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$

$\Rightarrow$ x = 1, y = 2, z = 3

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. On her birth day, Seema decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got ₹ 10 more. However, if there were 16 children more, everyone would have got ₹ 10 less. Let the number of children be x and the amount distributed by Seema for one child be y (in ₹).

Based on the information given above, answer the following questions:

(a) Represent the equations in terms x and y. (1)

(b) Write matrix equations to represent the information given above. (1)

(c) Find the number of children who were given some money by Seema. (1)

(d) How much amount is given to each child by Seema? (1)

Ans: (a) Here, number of children is x and amount distributed to one child is y (in ₹).

$\therefore$ Total money distributed $= xy$

According to the question, $(x-8)(y+10)=xy$

$\Rightarrow xy-8y+10x-80=xy$

$\Rightarrow 10x-8y=80$

$\Rightarrow 5x-4y=40$ …..(i)

and $(x+16)(y-10)=xy$

$\Rightarrow xy+16y-10x-160=xy$

$\Rightarrow -10x+16y=160$

$\Rightarrow 5x-8y=-80$ ….. (ii)

(b) We can write these equations in matrix form as $\begin{bmatrix}5 & -4\\ 5 & -8\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}40\\ -80\end{bmatrix}$

(c) Multiplying (i) 2 then subtracting with (ii), we get 5x = 160 $\Rightarrow$ x = 32

(d) Subtracting (i) and (ii), we get 4y = 120 $\Rightarrow$ y = ₹ 30

20. Three schools A, B and C decided to organize a fair for collecting money for helping the flood victims. They sold handmade fans, mats and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each respectively. The numbers of articles sold are given as

School /Article	A	B	C
Handmade fans	40	25	35
Mats	50	40	50
Plates	20	30	40



Based on the information given above, answer the following questions:
(a) What is the total money (in Rupees) collected by the school A?
(b) What is the total amount of money (in ₹) collected by schools B and C?
(c) What is the total amount of money collected by all three schools A, B and C?
(d) If the number of handmade fans and plates are interchanged for all the schools, then what is the total money collected by all schools?

Ans: Let x, y, z be the revenue collected by A, B and C.
The above problem can be represented in the form of matrix as

$$\begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix}\begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \begin{bmatrix} 1000+5000+1000 \\ 625+4000+1500 \\ 875+5000+2000 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \begin{bmatrix} 7000 \\ 6125 \\ 7825 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(a) Money collected by A = ₹ x = ₹ 7,000
(b) Money collected by B and C = ₹ $y + z$ = ₹ 6125 + 7825 = ₹ 14000
(c) Total amount collected by A, B and C = ₹ 7,000 + 14000 = ₹ 21000
(d) If the number of hand made fans and plates are inter changed for all the schools. then, Total hand fans made = 90
Total mats made = 140
Total plates made = 100
∴ Total money collected by all schools
= 90 × ₹ 25 + 140 × ₹ 100 + 100 × ₹ 50
= ₹ 2500 + ₹ 14000 + ₹ 5000 = ₹ 21,500